Rocked quantum periodic systems in the presence of coordinate-dependent friction

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We study rocked periodic systems with quantum fluctuations and coordinate-dependent friction. The steady current along the direction of the external driving force is evaluated numerically in terms of the path-integral Monte Carlo method and discussed in terms of the steepest descent approximation. Above the crossover temperature T_c a classical-like distribution function is given in order to calculate the least nonvanishing eigenvalue of the Fokker-Planck equation, thus one can finally obtain an expression of the current for moderate-to-strong damping. The influence of nonlinear quantum dissipation on the magnitude and the direction of the average current is observed. The classical current is also calculated using the Langevin simulation. The results show that when the friction is a periodic function of the coordinate, a net current averaging over the two directions arises even in the absence of both spatial and time asymmetry. [S1063-651X(98)10109-5]

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I. INTRODUCTION

A number of recent attempts to understand broad principles of directed transport have been focused on the overdamped ratchetlike periodic systems, which extract work out of the nonequilibrium fluctuations [1]. Here the driving force, the potential, and the noise in fact play cooperative roles. It is known that the combination of a ratchet and either colored noise [2] or a symmetric unbiased driving force [3], as well as the coupling between a symmetric periodic potential and a temporally asymmetric fluctuation [4], is enough to allow a net average current. More recently, the quantum rocked ratchet has been investigated in terms of the steepest descent approximation (SDA) [5]. The quantum effect becomes important for the transport properties in the regime of temperature $T \leqslant 2T_c$ and the tunneling can induce current reversal. In the SDA the barrier height is assumed to be large with respect to temperature k_BT, so it is necessary to improve this approach in the opposite region of temperatures $T > 2T_c$.

In the case of open systems, little work has been done taking into account the nonlinear quantum dissipation in the effective distribution function [6]. In particular, friction of the system near the well and that near the barrier may be quite different from each other; it has been shown that the introduction of coordinate-dependent friction can lead to qualitatively different physics [7,8]. For instance, its properties in either direction are different.

Here the path-integral Monte Carlo (PIMC) method is applied to the study of the quantum currents of a Brownian particle moving in the tilted washboard potentials [5]. The finite-height barrier and different dissipation mechanisms that exhibit a frequency dependence are included. The focus of this work is on the effects of nonlinear quantum dissipation on the current for moderate to strong damping; there exists a correction to the classical result. Moreover, we compare the results of the PIMC method with the one based on the Langevin simulation (LS) at high temperatures. We will show how space-dependent friction can induce the directed current even though both the ratchet potential and the time are completely symmetric. The direction of the average particle motion is controlled by the temperature as well as the parameters of the potential and friction.

II. MODEL AND METHOD

We introduce coordinate-dependent friction by allowing for a nonlinear coupling between the particle and the oscillator bath in the system-plus-reservoir Lagrangian

$$L = \frac{1}{2}m\dot{x}^2 - U(x,F) + \sum_{\alpha=1}^{\infty} \frac{m_\alpha}{2} \times \left\{ \dot{q}_\alpha^2 - \omega_\alpha^2 \left[q_\alpha - \frac{c_\alpha}{m_\alpha \omega_\alpha} f(x) \right]^2 \right\}. \tag{1}$$

Here the q_α's are harmonic bath modes, $U(x,F) = V(x) - Fx$ (cf. Fig. 1) is a tilted periodic potential, $V(x)$ denotes a periodic potential with period L, and F is an external driving force. The function $f(x)$ couples the bath modes nonlinearly to the particle coordinate x.

By eliminating the environmental degrees of freedom, the functional path-integral form of the partition function at the temperature $k_BT = 1/\beta$ can be expressed as [9]

$$Z(\beta) = \int \mathcal{D}[x(\tau)] \exp\{-S_{eff}[x(\tau)]/\hbar\}. \tag{2}$$

Here the effective action S_{eff} is given by

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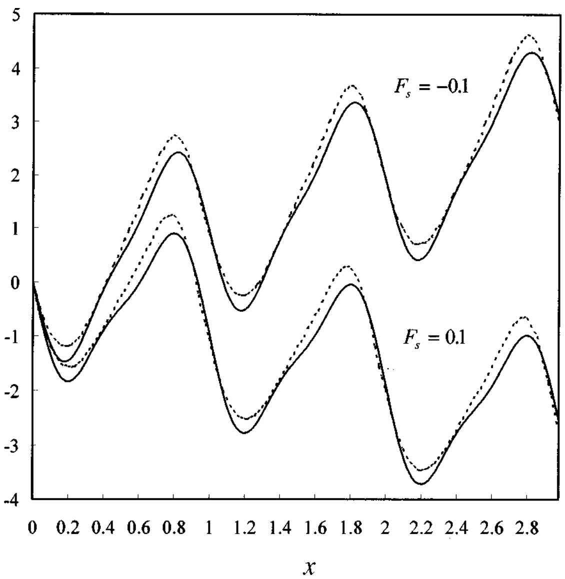

FIG. 1. Tilted ratchet U (solid lines) and its effective classical potential U_{eff} (dashed lines) when $F_s=-0.1$ (above) and $F_s=0.1$ (below).

$$S_{eff}[x(\tau)]=\int_0^{\hbar\beta}\left[\frac{1}{2}m\dot{x}^2(\tau)+U(x(\tau),F)\right]d\tau +\frac{1}{2}\int_0^{\hbar\beta}d\tau\int_0^{\hbar\beta}d\tau'\,\phi(\tau-\tau') \times[f(x(\tau))-f(x(\tau'))]^2. \tag{3}$$

The last term describes the influence of the environment and the influence kernel $\phi(\tau)$ is given by

$$\phi(\tau-\tau')=\frac{1}{2\pi}\int_0^{\infty}d\omega\,\theta(\omega)\frac{\cosh(0.5\hbar\beta\omega-\omega|\tau-\tau'|)}{\sinh(0.5\hbar\beta\omega)}. \tag{4}$$

Here $\theta(\omega)$ is the spectral density characterizing the bath, defined in Eq. (20).

Now we evaluate numerically the partition function using the Monte Carlo method. It is a generalized improvement of the Monte Carlo random walk algorithm [10] to a nonlinear quantum dissipation system. First we express the integral of the kinetic energy term in the form

$$Z(\beta)=\int dx_c\sqrt{\frac{m}{2\pi\hbar^2\beta}}\int_{-\infty}^{\infty}dx_1\int_{-\infty}^{\infty}dx_2\cdots\int_{-\infty}^{\infty}dx_{N-1} \times\left(\frac{Nm}{2\pi\hbar^2\beta}\right)^{(N-1)/2}\exp\Bigg\{-\frac{N}{2\hbar^2\beta}\sum_{n=1}^{N}(x_n-x_{n-1})^2 -\frac{\beta}{N}\sum_{n=1}^{N}U(x_n,F)-\frac{\hbar\beta^2}{2N^2}\sum_{n=1}^{N}\sum_{j=1}^{n}\phi\left[\frac{\hbar\beta}{N}(n-j)\right] \times[f(x_n)-f(x_j)]^2\Bigg\}, \tag{5}$$

where x_c is the coordinate of a classical trajectory. In order to make the contribution of the kinetic energy term to the exponent become a properly normalized Gaussian measure, it is also convenient to scale the new variables $\{\omega_n\}$ to have the range $[0,1]$. To do this, let us introduce the following recursion reaction for the kth random walk, x_n^k with $x_N^k=x_c$ $(n=1,2,\ldots,N-1;k=1,2,\ldots,M)$ [10,11]:

$$x_n^k=\frac{x_c}{N-n+1}+\frac{N-n}{N-n+1}x_{n-1}^k +\left[\frac{2\pi\hbar^2\beta}{m}\,\frac{N-n}{N(N-n+1)}\right]^{1/2}y(\omega_n^k), \tag{6}$$

where $y(\omega)$ is given by

$$\int_{-\infty}^{y_n}dy_n'\exp(-\pi y_n'^2)=\omega_n. \tag{7}$$

We use the rational approximation for $y(\omega)$ taken from Ref. [12].

Equation (5) then becomes

$$Z(\beta)=\int dx_c\sqrt{\frac{m}{2\pi\hbar^2\beta}}\exp[-\beta U(x_c,F)] \times\int_0^1 d\omega_1\int_0^1 d\omega_2\cdots\int_0^1 d\omega_{N-1} \times\exp\Bigg\{-\frac{\beta}{N}\sum_{n=1}^{N}[U(x_n,F)-U(x_c,F)] -\frac{\hbar\beta^2}{2N^2}\sum_{n=1}^{N}\sum_{j=1}^{n}\phi\left[\frac{\hbar\beta}{N}(n-j)\right][f(x_n)-f(x_j)]^2\Bigg\}. \tag{8}$$

Equation (8) as an (N-1)-dimensional average is straightforwardly evaluated using the Monte Carlo method. Thus the partition function is given by

$$Z(\beta)=\int dx_c\sqrt{\frac{m}{2\pi\hbar^2\beta}}\exp[-\beta U_{eff}(x_c,F)], \tag{9}$$

where the effective classical potential reads

$$U_{eff}(x_c,F)=U(x_c,F)-\frac{1}{\beta}\ln\Bigg\{\frac{1}{M}\sum_{k=1}^{M}\exp[-\beta U_k(x_c,F) -\beta G_k(x_c)]\Bigg\}, \tag{10}$$

with

$$U_k(x_c,F)=\frac{1}{N}\left[(1-N)U(x_c,F)+\sum_{n=1}^{N-1}U(x_n^k,F)\right], \tag{11}$$

$$G_k(x_c)=\frac{\hbar\beta}{2N^2}\sum_{n=1}^{N}\sum_{j=1}^{n}\phi\left[\frac{\hbar\beta}{N}(n-j)\right][f(x_n^k)-f(x_j^k)]^2. \tag{12}$$

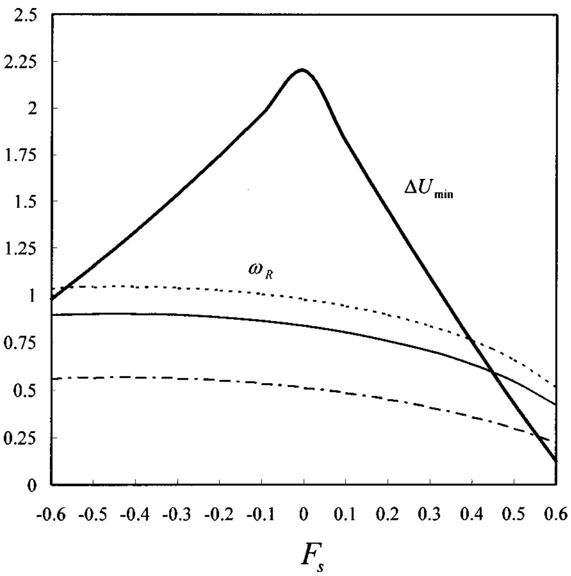

FIG. 2. Barrier height $\Delta U_{min}/\hbar\Omega_0$ (thick solid line, $\mu=1$) and the factor ω_R/Ω_0 (below three lines) as a function of the external driving force F_s. The parameters are $\alpha=1$, $\tau=1$ (thin solid line); $\alpha=1$, $\tau=2$ (dashed line); and $\alpha=2$, $\tau=1$ (dash-dotted line).

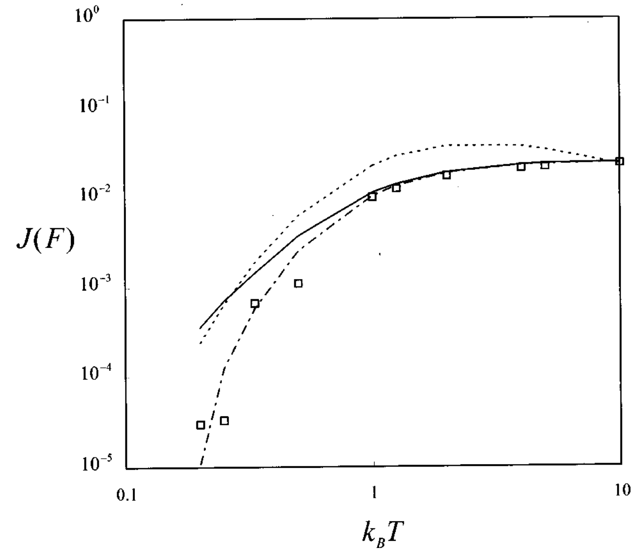

FIG. 3. Dependence of the current $J(F)$ on the temperature with the parameters $\alpha=1$ and $\tau=1$. The solid line shows the PIMC result, the dashed line shows the SDA result, the dash-dotted line shows the classical FP result, and the squares are the data of the LS.

We focus on the effective classical potential, which accounts for the effects of both quantum fluctuation and nonlinear dissipation, such that the stationary distribution function is given by (C is the normalization constant)

$$p(x_c,F)=C^{-1}\exp[-\beta U_{eff}(x_c,F)]. \tag{13}$$

Starting from Kramer's activation rate theory and the least nonvanishing eigenvalue of the Fokker-Planck equation above the crossover temperature, the probability current along the direction of the external driving force is related to the stationary distribution function by the relation

$$J(F)=\frac{\omega_R}{|U_b''|}\,\frac{k_BT[1-\exp(-FL/k_BT)]}{\int_0^L p(x,F)dx\int_x^{L+x}p(x',F)^{-1}dx'}. \tag{14}$$

Here ω_R denotes the coupling-induced dissipation and memory-renormalized barrier frequency, which also depends on F. It is determined by the largest positive root of the equation

$$\omega_R=\frac{\omega_b^2}{\omega_R+\frac{1}{m}\hat{\eta}(\omega_R)[f(x)/dx]_{x=x_b}^2}, \tag{15}$$

with $\omega_b=\sqrt{|U_b''|/m}$. Here $U_b''<0$ is the barrier's curvature of the tilted potential U at the extrema x_b and $\hat{\eta}(\omega_R)$ is the Laplace transform of the memory damping $\eta(t)$. The crossover temperature is defined by $T_c=\hbar\omega_R/2\pi k_B$. It is readily seen that the classical transmission factor $\omega_R/\omega_b<1$ determines the difference between the transition-state theory and the correct classical rate due to diffusive recrossing of the barrier.

This method is in fact not restricted to the classical regime because one can study as well the quantum probability distribution function of the system. Indeed, Eq. (14) has a correction to the overdamped cases [13] with the factor $\omega_R/|U_b''|$. In the case of weak thermal noise, the potential barrier is much larger than the temperature k_BT; then Eq. (14) can be reduced to the result based on the SDA [5]. On the other hand, at high temperatures, the effective classical potential U_{eff} reduces to the original potential U and thus Eq. (14) becomes a classical current. In the case of Ohmic damping $\hat{\eta}=\eta_0$, where η_0 is the zero-frequency friction coefficient, we have $\omega_R=(\sqrt{\eta_1^2+4m|U_b''|}-\eta_1)/2m$ with $\eta_1=\eta_0[df(x_b)/dx]^2$. For overdamped and large driving force cases, the factor $\omega_R/|U_b''|$ approaches a fixed value η_1^{-1}.

When the temperature is much less than the barrier height of the potential, the analytical expression for the current with a quantum fluctuation in terms of the steepest descent approximation is given by [5,9,14]

$$J_{qm}(F)=k_{qm}\frac{\omega_R}{2\pi}\frac{\sqrt{U_0''}}{\sqrt{|U_b''|}}\exp[-\beta\Delta U_{min}(F)] \times\{1-\exp(-\beta FL)\}, \tag{16}$$

with the quantum correction factor

$$k_{qm}=\prod_{n=1}^{\infty}\frac{mn^2\nu^2+n\nu\hat{\eta}(n\nu)\left[\frac{df(x)}{dx}\right]_{x=x_0}^2+U_0''}{mn^2\nu^2+n\nu\hat{\eta}(n\nu)\left[\frac{df(x)}{dx}\right]_{x=x_b}^2+U_b''} \tag{17}$$

and $\nu=2\pi k_BT/\hbar$. $\Delta U_{min}(F)$ denotes the smaller of the potential barriers $\Delta U(x,F)$ along the direction of the external driving force and U_0'' is the curvature of U at the ground state x_0. The factor k_{qm} tends to unity as $T\to\infty$ and diverges exactly at the crossover temperature T_c. It is worth noticing that the value of the quantum correction factor may be greater or less than unity if $df(x_b)/dx\neq df(x_0)/dx$.

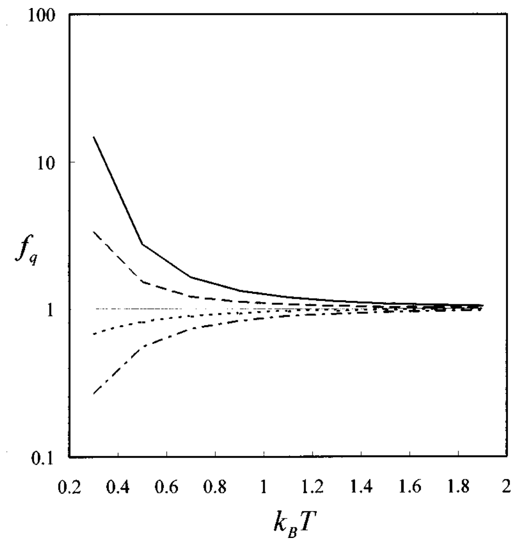

FIG. 4. Quantum correction factor as a function of the temperature for different $\lambda=-0.5$, 0.0, 0.5, and 0.8 from top to bottom. The parameters are $\mu=1$, $\alpha=2$, $\tau=1$, and $F_s=0.2$.

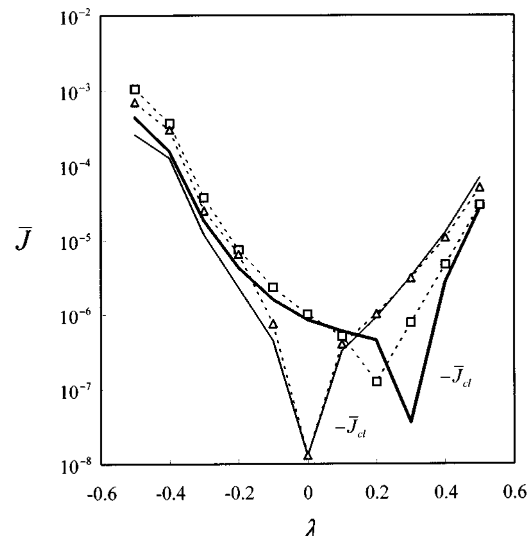

FIG. 5. Classical average current $\overline{J_{cl}}$ (solid lines) and its quantum correction $\overline{J_{qm}}$ (dashed lines) vs the nonlinear coupling constant λ for different $\mu=0$ (left) and 1 (right) at the temperature $k_BT=0.5\hbar\Omega_0$. Note that sign of the current changes on the right-hand side of the minima.

III. NUMERICAL CALCULATION OF QUANTUM CURRENTS

In the present paper the rocked periodic potential $U(x,F)$ and the coupling function $f(x)$ are taken in the forms

$$U(x,F)=V(x)-Fx=-V_0\left[\sin(2\pi x/L)+\frac{\mu}{4}\sin(4\pi x/L)\right]-Fx \tag{18}$$

and

$$f(x)=x+\frac{\lambda L}{2\pi}\cos(2\pi x/L), \tag{19}$$

where μ is an asymmetry parameter of the periodic potential $V(x)$ with $\mu=0$ and 1 denoting the symmetric and forward ratchets, respectively, and λ is a nonlinear coupling constant. A typical representative of a memory dissipation is the Drude form $\eta(t)=(\eta_0/\tau)\exp(-t/\tau)$. The corresponding $\theta(\omega)$ and $\hat{\eta}(\omega)$ read

$$\theta(\omega)=\frac{\eta_0\omega}{1+(\omega\tau)^2}, \quad \hat{\eta}(\omega)=\frac{\eta_0}{1+\omega\tau}. \tag{20}$$

Letting $\tau\to 0$ in Eq. (20), we recover the Ohmic damping case.

T, F, λ, τ, and a dimensionless parameter $\alpha=\eta_0/2m\Omega_0$ are five parameters of the model. Here $\Omega_0:=(2\pi/L)[V_0/m]^{1/2}$ is a frequency quantity introduced here to scale the current. The energy, e.g., the temperature k_BT, will be scaled in units of $\hbar\Omega_0$. The tilted potentials $U(x,\pm F)$ display a local maximum and minimum within each period; let us denoting by x_0 and $x_0^{\pm}$ one of the local minima and by x_b and $x_b^{\pm}$ its neighboring local maxima of $V(x)$ and $U(x,\pm F)$ for $F>0$. Those coordinates x_s satisfy the condition $\partial_x U(x_s,F)=0$, i.e.,

$$\cos\left(\frac{2\pi x_s}{L}\right)=\frac{\sqrt{1+2\mu^2-4\mu F_s}-1}{2\mu}, \tag{21}$$

and $F_s=FL/V_0 2\pi$. One thus has the relations $x_0^-<x_0<x_0^+$ and $x_b^-<x_b<x_b^+$ within one period structure for a tilted forward ratchet. The ground and barrier frequencies are determined by

$$\omega_0=\omega_b:=[|U''(x_s,F)|/m]^{1/2}=\Omega_0[|\sin(2\pi x_s/L)+\mu\sin(4\pi x_s/L)|]^{1/2}. \tag{22}$$

Hence $\omega_b^+<\omega_b<\omega_b^-$ due to $|U''(x_b^+)|<|U''(x_b)|<|U''(x_b^-)|$.

In the calculations of the PIMC method, we take $N=50$ and $M=1000$ to obtain sufficiently accurate results. Next we fix the barrier height of the original potential $V(x)$ through $V_0=1.5\hbar\Omega_0$. In Fig. 1 we plot the tilted washboard ratchets U and the effective classical potentials U_{eff} in units of $\hbar\Omega_0$ with the parameters $k_BT=0.5\hbar\Omega_0$, $\alpha=2$, $\tau=1$, and $\lambda=0.5$. We have found that the effect of quantum fluctuation makes the barrier height of U_{eff} decrease while the dissipation makes it increase.

The barrier height of the tilted periodic potential ΔU_{min} and the factor ω_R as a function of F_s are shown in Fig. 2. For an asymmetric periodic potential $V(x)$, ω_R is not an even function of the external driving force. It decreases with increasing $|F_s|$; this is because the frequencies of the ground and barrier states become small when $|F_s|$ increases. Also, ω_R decreases when α decreases and increases when τ decreases; thus α and τ will play opposing roles for the steady current. Moreover, we note that $\Delta U_{min}^+<\Delta U_{min}^-$ for the forward ratchets and then $J(F)\geqslant J(-F)$ for $F>0$ in the present of space-independent friction.

In the linear coupling case, i.e., $\lambda=0$, the temperature relation of the current $J(F)$ obtained from the various methods for $F_s=0.3$ is shown in Fig. 3. We observe that the

TABLE I. Average current $\overline{J}/\Omega_0$ vs temperatures k_BT for different external driving forces F_s.

	$\lvert F_s\rvert=0.1$		$\lvert F_s\rvert=0.2$		$\lvert F_s\rvert=0.3$	
k_BT	Classical	Quantum	Classical	Quantum	Classical	Quantum
0.2	0.983×10^{-9}	-3.299×10^{-9}	7.650×10^{-8}	-2.570×10^{-9}	1.236×10^{-6}	2.258×10^{-7}
0.3	-1.985×10^{-7}	-2.745×10^{-7}	1.016×10^{-6}	-2.652×10^{-7}	1.066×10^{-5}	2.174×10^{-6}
0.4	-3.434×10^{-6}	-3.518×10^{-6}	-2.755×10^{-6}	-6.344×10^{-6}	1.109×10^{-5}	-5.982×10^{-6}
0.5	-1.608×10^{-5}	-1.523×10^{-5}	-2.665×10^{-5}	-3.089×10^{-5}	-2.964×10^{-5}	-5.006×10^{-5}
0.6	-4.181×10^{-5}	-3.858×10^{-5}	-7.915×10^{-5}	-7.861×10^{-5}	-1.199×10^{-4}	-1.297×10^{-4}
0.7	-7.885×10^{-5}	-7.365×10^{-5}	-1.562×10^{-4}	-1.524×10^{-4}	-2.477×10^{-4}	-2.526×10^{-4}
0.8	-1.226×10^{-4}	-1.157×10^{-4}	-2.478×10^{-4}	-2.391×10^{-4}	-3.952×10^{-4}	-3.894×10^{-4}
0.9	-1.687×10^{-4}	-1.596×10^{-4}	-3.445×10^{-4}	-3.305×10^{-4}	-5.479×10^{-4}	-5.330×10^{-4}
1.0	-2.138×10^{-4}	-2.043×10^{-4}	-4.393×10^{-4}	-4.238×10^{-4}	-6.959×10^{-4}	-6.778×10^{-4}

current with quantum fluctuation can become enhanced up to one order of magnitude as compared to the current based on a pure classical analysis when the temperature is close to $2T_c$. It is seen from Fig. 3 that the SDA has a few errors when $k_BT>0.5\Delta U_{min}$; however, our resulting current of the PIMC method is in agreement with the LS at high temperatures.

The quantum correction factor is determined by the ratio of the quantum and classical currents in the calculations of the PIMC method, given by $f_q=J_{qm}/J_{cl}$. Indeed, k_{qm} appearing in Eq. (17) is an approximation of this quantity. The dependence of f_q on the temperature is plotted in Fig. 4 for different λ. It is interesting to see that the effects of nonlinear quantum dissipation make the values of the current increase or decrease. This behavior can be understood from Eq. (17): $k_{qm}\geqslant1$ when $df(x_0)/dx\geqslant df(x_b)/dx$, i.e., the friction of the system at the barrier is less than that of the ground state; however, k_{qm} may be less than unity when $df(x_0)/dx<df(x_b)/dx$, i.e., the friction of the system at the barrier is larger than that of the ground state.

The quantity of interest in the present paper is an average current $\overline{J}$: $\overline{J}=\frac{1}{2}[J(F)+J(-F)]$. Figure 5 plots the magnitude and direction of the average current $\overline{J}$ vs the nonlinear coupling constant λ for the tilted symmetric and forward ratchets. It is seen that the sign of the average current changes when λ varies even for a symmetric periodic potential with $\mu=0$. Based on the SDA, we know that the direction of $\overline{J}$ is determined by the three quantities $\omega_R/|U_b''|$, ΔU_{min}, and k_{qm}. Because the periodic potential has been tilted upward or downward, the positions of the local minima and maxima for $U(x,\pm F)$ are not identical and the values of the above three quantities are different along the two directions. We find that $\omega_R^-/|U''(x_b^-)|>\omega_R^+/|U''(x_b^+)|$ and $f_q^->f_q^+$; however, $\Delta U_{min}^-\geqslant\Delta U_{min}^+$ when $\lambda>0$. If the product of the former two quantities exceeds the latter, the current may be negative for both the symmetric and forward ratchets. Of course, the value of λ leading to the current reversal is reduced with decreasing μ.

The dependence of $\overline{J}$ on k_BT is shown in Table I with the parameters $\alpha=2$, $\tau=1$, $\mu=1$, and $\lambda=0.5$ for several values of F_s. When F_s decreases or the temperature k_BT increases, the asymmetric periodic potential plays less of a selective role. Thus $\overline{J}$ produces reversal when the temperature increases and the effect of quantum fluctuation makes the reversion of current easily realized at low temperatures. This behavior is also understood from Eqs. (14) and (16): $\Delta U_{min}^+<\Delta U_{min}^-$ and $\omega_R^+<\omega_R^-$ when $\mu=1$; however, the average current can be reversed if $f_q^+\ll f_q^-$. At high temperatures, the direction of the average current is determined by the factor $\omega_R/|U''|$ and $\lim_{T\to\infty}\overline{J}=|F|/2L(\omega_R^+/|U_b''^+|-\omega_R^-/|U_b''^-|)$, so it is possible that $\overline{J}<0$ when $\omega_R^-/|U''(x_b^-)|>\omega_R^+/|U''(x_b^+)|$ even for the forward ratchets.

IV. LANGEVIN SIMULATIONS OF CLASSICAL INERTIA RATCHET

The generalized Langevin equation with space- and time-dependent friction can be derived from Eq. (1) and is given by [15,16]

$$m\ddot{x}(t)+\int_0^t ds\,\frac{df(x(t))}{dx(t)}\,\eta(t-s)\frac{df(x(s))}{dx(s)}\dot{x}(s)+\frac{\partial U(x(t),F)}{\partial x(t)}=\frac{df(x(t))}{dx(t)}\varepsilon(t). \tag{23}$$

The Gaussian random force $\varepsilon(t)$ has zero mean and obeys the generalized fluctuation dissipation theorem,

$$\langle\varepsilon(t)\varepsilon(s)\rangle=k_BT\eta(t-s). \tag{24}$$

Now the friction kernel function $k(t)$ is taken to be the Ohmic δ-correlated form in order to study mainly the effects of the coordinate-dependent friction on the inertia ratchets. The equations of motion for the particle with mass $m=1$ then read

$$\dot{x}(t)=v(t),$$
$$\dot{v}(t)=h(x,v)+g(x)\xi(t)+F(t), \tag{25}$$

with

$$h(x,v)=-\eta(x)v(t)-\frac{dV(x)}{dx},\quad g(x)^2=\eta(x). \tag{26}$$

The newly defined random force $\xi(t)$ is Gaussian, has zero mean, and most importantly is δ correlated,

$$\langle\xi(t)\rangle=0,\quad\langle\xi(t)\xi(t')\rangle=2k_BT\delta(t-t'). \tag{27}$$

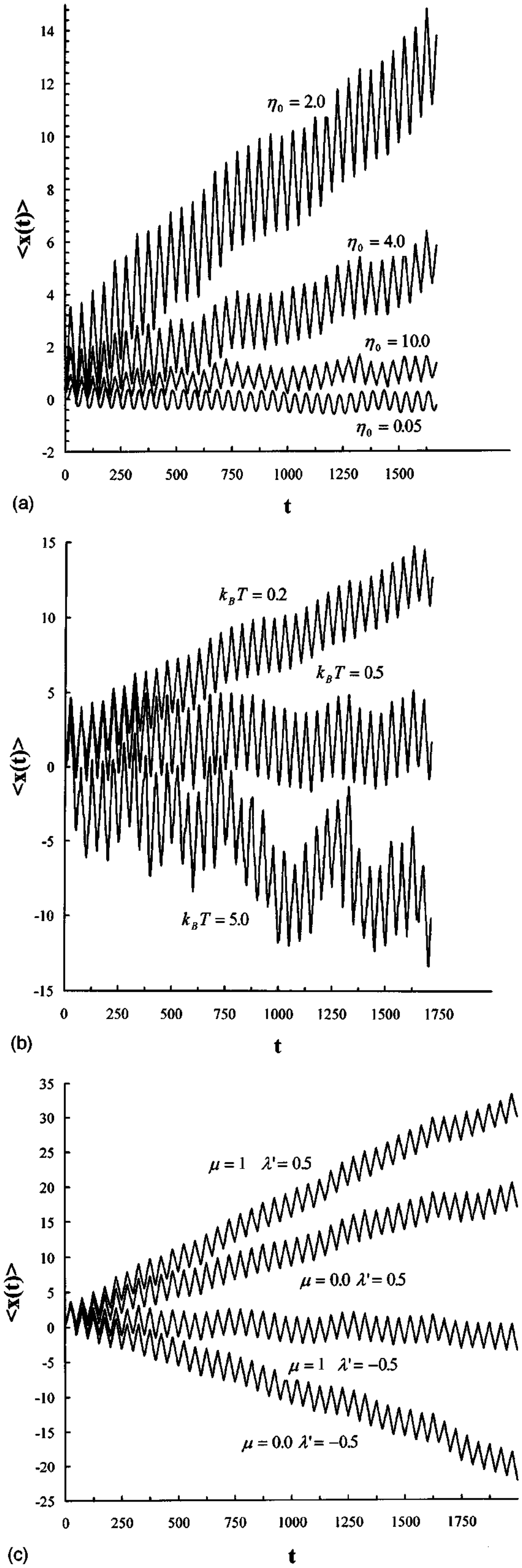

FIG. 6. Average position $\langle x(t)\rangle$ vs time (the common parameters are $t_e = 50.0$ and $A = 0.5$) as a function of (a) η_0 with $\mu = 1$, $\lambda' = 0.0$, and $k_BT = 0.2$ (here the results corresponding to $\eta_0 = 0.05$ are reduced by two orders of magnitude), (b) k_BT with $\mu = 1$, $\lambda' = 0.0$, and $\eta_0 = 2.0$, and (c) the parameters μ and λ' with $k_BT = 0.2$ and $\eta_0 = 2.0$.

Here we will again consider space-dependent friction, which is a symmetric periodic function with the same period as the potential

$$\eta(x) = \eta_0[1 + \lambda' \cos(2\pi x/L)]. \tag{28}$$

For external force we use a systematic deterministic square-wave field with zero mean in time:

$$F(t) = A, \quad 0 \leqslant t < \frac{1}{2} t_e, \quad \bmod t_e,$$

$$F(t) = -A, \quad \frac{1}{2} t_e < t \leqslant t_e, \quad \bmod t_e. \tag{29}$$

t_e is the time period of the external driving force, which is assumed to be longer than any other time scale of the system in the adiabatic limit.

In this section we apply the stochastic Runge-Kutta algorithm [17] to numerically integrate a set of Langevin equations (25). In order to find out whether or not a current exists, we shall look for the behavior of the average position $\langle x(t)\rangle$ of the particle as a function of time. In Figs. 6(a)–6(c) the average position of the particle driven by an external square-wave field in the presence of space-dependent friction is shown. Every situation described below was obtained with 100 realizations and $\Delta t = 0.01$ time step. The different curves correspond to different damping coefficients η_0 [Fig. 6(a)], to different temperatures k_BT [Fig. 6(b)], and to different nonlinear friction constants λ' in Eq. (28) [Fig. 6(c)].

We want to make the following points. First, in Fig. 6(a), there are no transitions out of the wells when $\eta_0 \to \infty$ and therefore no current. If the friction is too small, i.e., $\eta_0 \to 0$, there is also no current, although the oscillation of the particle along the two directions is strong. Hence a finite, stationary current requires a finite dissipation for the rocked ratchets. Next, in Fig. 6(b) we observe that the current vanishes when $k_BT \to 0$ unless A is quite large. However, the current does not vanish and reverses when k_BT is large, as discussed in Sec. III. That is to say that the strong noise does not completely eliminate the effects of the potential asymmetry. Finally, in Fig. 6(c), it is important to note the behavior of the current as a function of λ' for fixed k_BT and η_0. Note that the direction of the current varies with λ' because the friction at the barriers is different for the upward and downward tilted ratchets.

V. CONCLUSION

We apply the path-integral Monte Carlo method to study the probability current of the one-dimensional rocked ratchet systems with quantum fluctuation and space-dependent memory damping. Above the crossover temperature, a net average current is induced by a zero-mean external driving force in the presence of coordinate-dependent friction even for the symmetric periodic potential. The present approach can be reduced to the classical current with finite barrier at high temperatures and agrees with the result of the steepest descent approximation at low temperatures. The quantum effect makes a ratchet play less of a selective role and the memory time of the damping plays an opposing role for the current against the friction strength. The coordinate-dependent friction can make the magnitude of the quantum current along the direction of the external driving force increase or decrease. Using the Langevin simulations for the classical inertia ratchet with space-dependent friction, we find that the particle can reverse the direction of the average motion upon a variation of the temperature or depending on the form of friction.

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